Convergence of Series I - Answer Key

1. Find the nth partial sum for the series $3+2-2+2-2+\ldots$.

   $S_n = 5$ for n odd and 3 for n even.

2. Does the series $3+2-2+2-2+\ldots$ converge? Why?

   No, because the sequence of partial sums does not converge.

3. Does the series $6+3+\frac{3}{2}+\frac{3}{2^2}+\ldots$ converge? If so, find the sum.

   $6\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = 12.$

4. Does the series $-5+\frac{5}{2}-\frac{5}{3}+\frac{5}{4}-1+\ldots$ converge? If so, find the sum.

   $-5\ln(2)$

5. Does the series $\sum_{n=0}^{1,578,993,21} ne^{n^2}$ converge? Why?

   Yes, because the series in finite.

6. Does the series $\sum_{n=10}^{\infty}\frac{3n^3+2}{2n^3-1}$ converge? Why?

   No, by the divergent test.

7. Find the mth partial sum for the series $\sum_{n=2}^{\infty}\frac{1}{n^2-1}$.

   $S_m = \frac{3}{4}-\frac{1}{2m}-\frac{1}{2m+2}$.

8. Does the series $\sum_{n=2}^{\infty}\frac{1}{n^2-1}$ converge? Why?

   Yes, because $\lim_{m\to\infty} S_m = \frac{3}{4}$.

9. Does $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2} + e^{-n}$ convergent? Why?

   Yes, because both $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ and $\sum_{n=1}^{\infty}e^{-n}$ are convergent.

10. Does the series $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$ converge? Why?

    No, by the divergent test.